

# New Implications of Lorentz Violation

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In this proceedings, I summarize two recently discovered theoretical implications that Lorentz violation has on physical systems. First, I discuss new models for neutrino oscillations in which relatively simple combinations of Lorentz-violating parameters can mimic the major features of the current neutrino oscillation data. Second, I will present results on Yang-Mills instantons in Lorentz-violating background fields. An explicit solution is presented for unit winding number in  $SU(2)$ .

## 1 Introduction

Enormous success in particle physics has been obtained during the last century by assuming symmetry of the fundamental action under the Lorentz group. Supplementing this with various assumed gauge symmetries and representational content eventually led to the standard model. A key step in constructing the standard model involves spontaneously breaking one of these assumed symmetries as well as relaxing some of the discrete symmetries in the electroweak sector. A natural question arises as to the validity of perfect symmetry under the Lorentz group as well.

In fact, there are theoretical reasons to suspect that Lorentz symmetry breaking may arise naturally in more fundamental theories such as string theory[1] or other attempts at quantum gravity[2]. In addition, there are numerous experimental tests of Lorentz invariance in a variety of sectors[3]. A general framework for including general Lorentz breaking effects into the standard model has been constructed[4, 5]. The resulting effective field theory is called the Standard Model Extension (SME). Stability and causality issues as well as generic properties of the dispersion relations have also been studied[6].

## 2 Lorentz and CPT Violation

For about the past fifteen years, it has been known that miniscule remnant effects that violate Lorentz invariance may arise in a more fundamental the-

ory of nature[1]. In addition, the well known CPT theorem proves that any local, Lorentz invariant quantum field theory must also preserve CPT. In fact, this theorem has been expanded to prove that CPT violation implies Lorentz violation[10], demonstrating that bounds on CPT can be interpreted as bounds on Lorentz violation.

The generic features of such violations may be incorporated into effective field theory using a generic spontaneous symmetry breaking mechanism that is analogous to the conventional Higg's mechanism of the standard model. The crucial difference concerns the field that exhibits a nonzero vacuum expectation value. In conventional Higg's models, the field that is used to break electroweak symmetry is taken as a scalar field in order to preserve Lorentz invariance as well as renormalizability. Consider a generic field theory containing gauge bosons with tensor indices ( $B^\mu$  for example) with nontrivial couplings to the fermions (terms of the type  $B^\mu \bar{\psi} \gamma^5 \gamma_\mu \psi$  for example). A Lorentz covariant potential for the tensor field can induce a nonzero expectation value of the form  $\langle B^\mu \rangle$  that will generate Lorentz-violating contributions to the matter sectors.

The SME consists of all possible terms that couple the standard model fields to background tensor fields.[4, 5] It is the spirit of the model to be as general as possible so that any experiment that exhibits Lorentz violation in the future can be described in this formalism. The hope is that experimentally identifying specific constants for Lorentz violation that occur in nature may serve as a window to a more fundamental theory. On the theoretical side, the SME is general enough to accommodate any theory that involves Lorentz Violation. For example, it has been argued that any realistic theory of noncommutative geometry must reduce to a subset of the SME[11]. For practical calculations, it is often useful to restrict the couplings to a minimal set that preserves the conventional gauge invariance of the standard model as well as power counting renormalizability. Imposing translational invariance on these couplings yields the minimal SME, useful for quantifying leading order corrections to experiments.

As an example, consider the electron-photon sector. Imposing gauge invariance and restricting to power-counting renormalizable terms in the standard model extension yields a lagrangian of

$$L = \frac{1}{2} i \bar{\psi} \Gamma^\nu \overleftrightarrow{D}_\nu \psi - \bar{\psi} M \psi + L_{\text{photon}} \quad , \quad (1)$$

where  $\Gamma$  and  $M$  denote

$$\Gamma^\nu = \gamma^\nu + c^{\mu\nu} \gamma_\mu + d^{\mu\nu} \gamma_5 \gamma_\mu \quad , \quad (2)$$

$$M = m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu} \quad . \quad (3)$$

The parameters  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $H$  are related to fixed background expectation values of tensor fields. In this sector, stringent bounds on many parameters have been attained. For example, limits on the order  $|k| < 10^{-32}$  for photons[12], and  $|b_3| < 10^{-24} m_e$  for electrons[13] have been obtained.

Different sectors of the SME have independent parameters for the background fields, therefore the stringent limits in electrodynamics do not rule out

potentially large effects in other sectors. For example, as I will discuss next, it may be possible that current experimental data regarding neutrino oscillations can be modeled using Lorentz violating terms, rather than masses.

### 3 Application to Neutrino Oscillations

The conventional formalism appears to describe much of the current data involving neutrino masses fairly well using mass differences on the order of  $\Delta m^2 \sim 10^{-20} \text{ GeV}^2$ . The ratio  $\Delta m^2/E^2 \sim 10^{-20}$  happens to be compatible with leading order Planck suppression estimates of the Lorentz-Violation parameters. It is therefore reasonable to ask if these oscillation effects are really a manifestation of Lorentz-violating background fields coupled to neutrinos.

The SME effective hamiltonian for neutrinos and antineutrinos in the presence of Lorentz violation has recently been constructed[5, 8]. This model is important as it includes all possible leading order corrections to the neutrino propagators in the presence of Lorentz violation. All previous work on neutrinos in the presence of Lorentz violation has assumed a rotationally invariant subset of the SME (called Fried Chicken models) typically with two neutrino species and nonzero neutrino masses[7]. The general case that includes three neutrino species and allows for violation of rotational symmetry can be expressed using an effective hamiltonian in the active neutrino basis  $(\nu_a, \bar{\nu}_a)$ , where  $a$  represents  $e$ ,  $\mu$ , or  $\tau$ .

$$(h_{eff})_{ab} = |\vec{p}| \delta_{ab} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2|\vec{p}|} \begin{pmatrix} (\tilde{m})_{ab} & 0 \\ 0 & (\tilde{m})_{ab}^* \end{pmatrix} + \frac{1}{|\vec{p}|} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \quad (4)$$

where

$$M_{11} = [(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab} \quad , \quad (5)$$

$$M_{12} = -i\sqrt{2}p_\mu (\epsilon_+)^\nu [(g^{\mu\nu\sigma} p_\sigma - H^{\mu\nu})C]_{ab} \quad , \quad (6)$$

$$M_{21} = i\sqrt{2}p_\mu (\epsilon_+)^*\nu [(g^{\mu\nu\sigma} p_\sigma + H^{\mu\nu})C]_{ab}^* \quad , \quad (7)$$

$$M_{22} = [-(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab}^* \quad . \quad (8)$$

For illustrative purposes, this form can be restricted to the minimal SME[5] for which only the left-handed neutrino doublet  $L_a$  is present. The resulting lagrangian contains the terms

$$L \supset \frac{i}{2} \bar{L}_a \gamma^\mu \overleftrightarrow{D}_\mu L_a - (a_L)_{\mu ab} \bar{L}_a \gamma^\mu L_b + \frac{i}{2} (c_L)_{\mu\nu ab} \bar{L}_a \gamma^\mu \overleftrightarrow{D}_\nu L_b \quad , \quad (9)$$

yielding the effective neutrino hamiltonian

$$(h_{eff})_{ab} = |\vec{p}| \delta_{ab} + \frac{1}{|\vec{p}|} [(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab} \quad . \quad (10)$$

Note that  $c_L$  ( $a_L$ ) preserves (violates) CPT. Diagonalization of this matrix yields two momentum dependent eigenvalue differences that govern the neutrino and antineutrino oscillation probabilities.

Some generic features of these oscillation probabilities may be identified by analyzing dimensionless combinations of parameters that appear in the oscillatory function arguments. For the standard massive neutrino case, the relevant ratio is  $\Delta m^2 \cdot (L/E)$ . The Lorentz violation terms typically contribute  $a_L \cdot (L)$  and  $c_L \cdot (LE)$  factors in the argument. This means that novel new energy dependences for the oscillations may be attained. In general, there will also be rotationally noninvariant terms contributing to the oscillation arguments. This opens up the possibility for interesting searches for diurnal variations at the Earth's sidereal period  $\omega \approx 23 \text{ h } 56 \text{ m}$ . A realistic model within the minimal SME that appears consistent with current experimental data is the bicycle model[8]. This model is notable since it consists of a two parameter fit to the currently observed data, while at the same time maintaining the full gauge invariance of the standard model. The bicycle model sets all Lorentz violating parameters to zero, except the rotationally invariant piece of  $c_L$  and a single spatial component of  $a_L$ .

Regardless of the specific choice of parameters, there are specific signatures for Lorentz Violation in neutrino oscillations. They are:

- Spectral anomalies (L or L/E oscillation behavior).
- L - E conflicts for experiments in different regions of L - E space that cannot be accommodated using only two mass differences.
- Periodic Variations, such as a diurnal signal.
- Compass asymmetries (effects that cannot be attributed standard physics such as the effect of the Earth's magnetic field on cosmic rays).
- Neutrino-antineutrino mixing.
- Classic CPT test:  $P_{\nu_b \rightarrow \nu_a} \neq P_{\bar{\nu}_a \rightarrow \bar{\nu}_b}$ .

Note that the only one of these for which there is a possible signal is the L - E conflict of LSND[14] to be tested by the future data collected by MiniBooNE.

## 4 Yang-Mills Instantons with Lorentz Violation

Static solutions to pure Yang-Mills theories in four Euclidean dimensions are well known and are called instantons. The pure Yang-Mills sector of the SME contains terms that violate the Lorentz symmetry, but it turns out that many of the properties of instanton solutions remain intact[9]. This result is due to the fact that the instanton solutions rely heavily upon topological arguments as will be discussed in the remainder of this proceedings.

The standard pure Yang-Mills Euclidean action is given by

$$S_0(A) = \frac{1}{2} \int d^4x \text{Tr}[F^{\mu\nu} F^{\mu\nu}] \quad , \quad (11)$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu] \quad , \quad (12)$$

is the curvature of the connection  $A$ . The topological charge  $q$  is defined as

$$q = \frac{g^2}{16\pi^2} \int d^4x \text{Tr} \tilde{F}^{\mu\nu} F^{\mu\nu} \quad , \quad (13)$$

where  $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^{\alpha\beta}$  is the dual of  $F$ . A useful identity is:  $\frac{1}{4}\text{Tr}\tilde{F}F = \partial^\mu X^\mu$ , with

$$X^\mu = \frac{1}{4}\epsilon^{\mu\nu\lambda\kappa}\text{Tr}(A^\nu F^{\lambda\kappa} - \frac{2}{3}igA^\nu A^\lambda A^\kappa) \quad . \quad (14)$$

This converts the topological charge integral to a surface integral with the net result that  $q$  must be an integer. Note that this argument is independent of the specific form of the action.

The equation of motion for the curvature is

$$[D^\mu, F^{\mu\nu}] = 0 \quad , \quad (15)$$

with a corresponding Bianchi Identity that follows from the definition of  $F$ :

$$[D^\mu, \tilde{F}^{\mu\nu}] = 0 \quad . \quad (16)$$

( $D_\mu = \partial_\mu + iqA_\mu$  is the usual covariant derivative) This gives a set of nonlinear differential equations for  $A^\mu$ . A clever argument for solving[15] these equations involves consideration of the inequality

$$\frac{1}{2} \int d^4x \text{Tr}(F \mp \tilde{F})^2 \geq 0 \quad . \quad (17)$$

This can be rearranged as

$$S \geq \pm \frac{1}{2} \int d^4x \text{Tr}[\tilde{F}^{\mu\nu} F^{\mu\nu}] = \pm \frac{8\pi^2}{g^2} q \quad . \quad (18)$$

The inequality is saturated for  $F = \pm \tilde{F}$ , implying that self-dual or anti-self-dual curvatures are extremal solutions.

As an example of an explicit self-dual solution, let  $q = 1$ , with gauge group  $G = \text{SU}(2)$ . The vector potential can be written as

$$A_{SD}^\mu = -\frac{\tau^{\mu\nu}x^\nu}{g(\rho^2 + x^2)} \quad , \quad (19)$$

and the corresponding curvature is

$$F_{SD}^{\mu\nu} = \frac{2\rho^2}{g(\rho^2 + x^2)^2} \tau^{\mu\nu} \quad , \quad (20)$$

where  $\tau^{0i} = \sigma^i$  and  $\tau^{ij} = \epsilon^{ijk}\sigma^k$  are written in terms of the conventional Pauli sigma matrices. The free parameter  $\rho$  controls the instanton size. The anti-self-dual solution ( $q = -1$ ) is obtained using the parity transform of the above solution. Subsequently, all self-dual solutions were classified[16].

Next, the Lorentz-violating case is examined. The quadratic action that preserves gauge invariance is given by

$$S(A) = \frac{1}{2} \int d^4x \text{Tr}[(F^{\mu\nu} F^{\mu\nu}) + (k_F)^{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}] \quad , \quad (21)$$

where the parameters  $k_F$  are small, constant background fields. Only terms of  $O(k_F)$  are kept in the calculations. The first result is that the topological charge  $q$  remains integral because the conventional argument is insensitive to the detailed form of the action, provided that gauge invariance is maintained. A modified bound on the action is

$$S \geq \pm \frac{8\pi^2}{g^2} q \pm \frac{1}{4} (k_F^{\mu\nu\alpha\beta} + \tilde{k}_F^{\mu\nu\alpha\beta}) \int d^4x \text{Tr} \tilde{F}^{\mu\nu} F^{\alpha\beta} \quad , \quad (22)$$

where  $\tilde{k}_F^{\mu\nu\alpha\beta} = \frac{1}{4} \epsilon^{\mu\nu\lambda\kappa} k_F^{\lambda\kappa\rho\sigma} \epsilon^{\rho\sigma\alpha\beta}$ . It is useful to decompose  $k_F = k_F^+ \oplus k_F^-$  according to its duality properties and consider the two cases separately.

For case one ( $k_F = -\tilde{k}_F$ ), the background constants take the form  $k_F^{\mu\nu\alpha\beta} = \Lambda^{\mu[\alpha} \delta^{\nu]\beta}$ , where  $\Lambda^{\mu\nu}$  is a symmetric, traceless matrix. The action is then extremal for the modified duality condition

$$F' \simeq \pm \tilde{F}' \quad , \quad (23)$$

where  $F'^{\mu\nu} = F^{\mu\nu} + \frac{1}{2} k_F^{\mu\nu\alpha\beta} F^{\alpha\beta}$ . Explicit solutions are constructed using  $\tilde{x}^\mu = x^\mu + \Lambda^{\mu\nu} x^\nu$ , and the vector potential is given by  $A^\mu(x) \simeq A_{SD}^\mu(\tilde{x}) + \Lambda^{\mu\nu} A_{SD}^\nu(x)$ . These solutions take the form of conventional instantons in skewed coordinates.

For case two ( $k_F = \tilde{k}_F$ ), the background constants are trace free. In this case, the lower bound on  $S$  given by (22) varies with  $\delta F$ . This means that the previous modified duality condition fails to generate a solution and the equations of motion must be solved explicitly. This can be done to leading order in  $k_F$  by expanding  $A = A_{SD} + A_k$ , fixing  $A$  to be close to the conventional self-dual solution. The equations of motion become

$$[D_{SD}^\nu, [D_{SD}^\nu, A_k^\mu]] + 2ig[F_{SD}^{\mu\nu}, A_k^\nu] = j_k^\mu \quad , \quad (24)$$

where  $j_k^\mu \equiv k_F^{\mu\nu\alpha\beta} [D_{SD}^\nu, F_{SD}^{\alpha\beta}]$ .

This is a second-order, linear elliptic differential equation. A formal solution can be constructed using the relevant propagator  $G(x, y)$ :

$$A_k = \int d^4y G(x, y) j_k(y) \quad (25)$$

For the case  $q = 1$  with  $G = \text{SU}(2)$ , an explicit solution can be constructed using the following procedure:

- Transform to singular gauge  $\rightarrow$  makes fields  $\sim O(\rho^2)$ .
- To  $O(\rho^2)$  in this gauge, can use free propagator

$$G_0(x, y) = \frac{1}{4\pi^2(x - y)^2} \quad .$$

- Gives tensorial structure for general ansatz

$$A_k^\mu = \frac{2\rho^2 x^2}{3g} f(x^2) k_F^{\mu\nu\alpha\beta} \tau^{\alpha(\nu} x^{\beta)} \quad .$$

- Substitute into the full equation of motion with general  $\rho$ .

Remarkably this gives a differential equation for  $f(x)$ , indicating that the tensorial structure is in fact correct to all orders in  $\rho^2$ .

## 5 Conclusion

A general formalism allowing for Lorentz violation (and possible resulting CPT violation) in the neutrino sector has been developed. Possible signals for Lorentz violation include anomalous energy dependence as well as sidereal variations. To date, only a tiny subset of the neutrino sector implications have been explored. In addition, it has been shown that instantons can still be classified according to the conventional topological charge.

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